

Arrow of Time in Rigged Hilbert Space Quantum Mechanics

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Arno Bohm and Ilya Prigogine's Brussels–Austin Group have been working on the quantum mechanical arrow of time and irreversibility in rigged Hilbert space quantum mechanics. A crucial notion in Bohm's approach is the so-called preparation/registration arrow. An analysis of this arrow and its role in Bohm's theory of scattering is given. Similarly, the Brussels–Austin Group uses an excitation/de-excitation arrow for ordering events, which is also analyzed. The relationship between the two approaches is initially discussed focusing on their semi-group operators and time arrows. Finally a possible realist interpretation of the rigged Hilbert space formulation of quantum mechanics is considered.

KEY WORDS: quantum mechanics; arrow of time; irreversibility; rigged Hilbert space.

1. INTRODUCTION

When Dirac introduced his formalism for quantum mechanics (Dirac, 1930/1981), it lacked a rigorous mathematical foundation. von Neumann's pioneering work on Hilbert space theory (von Neumann, 1932/1955) became the mathematical foundation for quantum mechanics (QM). Nevertheless, many physicists preferred using Dirac's bra-ket formalism because of its calculational convenience among many other advantages including: (1) observables can be treated as continuous operators, (2) Hermitian observables have a complete set of eigenkets and their corresponding eigenvalues can be discrete or continuous, and (3) state vectors are well-behaved smooth functions. However, a rigorous justification for Dirac's formalism cannot be given within Hilbert space (HS).

There are additional reasons to extend the HS formulation of QM to a broader mathematical framework such as a rigged Hilbert space (RHS), also known as a Gel'fand triplet or equipped space (Gel'fand and Vilenkin, 1964; Gel'fand and

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Shilov, 1967; Bohm, 1967; Bohm and Gadella, 1989; Nagel, 1989). After briefly reviewing RHS (Section 2), some reasons for going beyond HS will be given, particularly scattering and decay phenomena (Sections 3–6). Finally, some initial thoughts toward a realist interpretation of RHS QM are considered (Sections 7–8).

2. WHAT IS RHS?

Let Ψ be an abstract linear scalar product space and complete Ψ with respect to two topologies. The first topology is the standard HS topology $\tau_{\mathcal{H}}$ defined by the norm

$$\|h\| = \sqrt{(h, h)} \quad (1)$$

where h is an element of Ψ . The second topology τ_{Φ} is defined by a countable set of norms

$$\|\phi\|_n = \sqrt{(\phi, \phi)_n}, \quad n = 0, 1, 2, \dots \quad (2)$$

where ϕ is also an element of Ψ and the scalar product in (2) is given by

$$(\phi, \phi')_n = (\phi, (\Delta + 1)^n \phi'), \quad n = 0, 1, 2, \dots \quad (3)$$

where Δ is the Nelson operator $\Delta = \sum_i \chi_i^2$. The χ_i are the generators of an enveloping algebra of observables for the system in question and they form a basis for a Lie algebra (Nelson, 1959; Bohm *et al.*, 1999). For example if we are modeling the harmonic oscillator, the χ_i would be the position and momentum operators or, alternatively, the raising and lowering operators (Bohm, 1978). Furthermore if the operator $\Delta + 1$ is *nuclear* then the space Φ defined by (2) is a nuclear space (Bohm, 1967; Treves, 1967).

We obtain a Gel'fand triplet by completing Ψ with respect to τ_{Φ} to obtain Φ and with respect to $\tau_{\mathcal{H}}$ to obtain \mathcal{H} . In addition we consider the dual spaces of continuous linear functionals Φ^{\times} and \mathcal{H}^{\times} respectively. Since \mathcal{H} is self dual, we obtain

$$\Phi \subset \mathcal{H} \subset \Phi^{\times}. \quad (4)$$

The Nelson operator fully determines the space Φ . However, there are many inequivalent irreducible representations of an enveloping algebra of a group characterizing a physical system (Bohm *et al.*, 1999). Therefore further restrictions may be required to obtain a realization for Φ . The particular characteristics of the physical context of the system being modeled provide some restrictions analogous to the situation for GNS representations for the construction of W^* -algebras in algebraic quantum mechanics. Additional restrictions may be required due to the convergence properties desired for test functions in Φ . In general, one chooses the weakest topology such that the algebra of operators for the physical problem

is continuous and Φ is nuclear. The physical symmetries of the system play an important role in such choices (Bohm *et al.*, 1999).³

In many regards, working in a RHS is only as complicated as using standard Dirac bra-ket notation. There is an additional conceptual problem introduced in the RHS extension to QM that is not present in the ordinary HS formulation; namely, the choice of riggings Φ and Φ^\times are problem dependent. Every physical system, or at best classes of systems as in scattering, has its own RHS distinguished by the algebra of observables. This problem does not exist in HS where the natural norm topology is prescribed for all physical systems.

The typical choice for a realization of HS is the space of equivalence classes of Lebesgue square integrable functions L^2 . Smooth functions are defined for every point, but the equivalence classes of L^2 functions, the vectors of the HS, are not so defined. In RHS there are no equivalence class problems. The vectors of Φ are functions that are defined point-wise and are typically Riemann integrable. While the position and momentum operators do not have eigenvectors in HS (Gel'fand and Shilov, 1967), all eigenstates are well defined in RHS.⁴

More generally, RHS contains observables with continuous or even complex eigenvalues, whereas HS does not, because the dual space Φ^\times contains the appropriate eigenvectors along with distributions. This means that the basis vector expansion of eigenvectors (Dirac's spectral decomposition) can be given a rigorous foundation resulting in the nuclear spectral theorem:

$$|\phi\rangle = \sum_n |E_n\rangle\langle E_n|\phi\rangle + \int |E\rangle\langle E|\phi\rangle d\mu(E). \tag{5}$$

Here the rounded bras and kets denote elements definable on HS and the first term in (5) represents the discrete part of the spectrum. The angular bras and kets $\langle E|$, $|E\rangle$ denote elements defined in Φ^\times , so the second term in (5) represents the continuous part of the spectrum.

3. EXTRINSIC VERSUS INTRINSIC IRREVERSIBILITY

Solutions of Schrödinger's equation in HS describe the temporal evolution of isolated quantum systems in a time-reversible manner, yet many quantum systems

³ In the simple example of the harmonic oscillator, choosing the raising and lowering operators as the generators for the algebra or the position and momentum operators as the generators would yield different Nelson operators, but the results are physically equivalent. However in general one does not get physically equivalent results (e.g. choosing a different value of j in the rotation group corresponds to a different physical system/situation). So one has to look at the symmetries, boundary conditions, causal mechanisms, etc. in order to decide which representation of an enveloping algebra to use as a representation.

⁴ Hence, in RHS the observables form an algebra on the entire space of physical states (including Φ^\times , where Dirac kets reside).

exhibit irreversible behavior (e.g. resonance and decaying states). There are two distinct ways of describing irreversible processes (e.g. Atmanspacher *et al.*, 2002). Irreversible behavior in quantum systems is usually viewed as solely due to the interaction of a system with its environment. This approach to irreversibility is described as *extrinsic*, because the environment is crucial for irreversible evolution. Examples of extrinsic irreversibility are given by any open-system described by a master equation. By contrast *intrinsic* irreversibility refers to irreversible behavior generated by the dynamics of a system; that is to say, the Hamiltonian describes this irreversibility without explicit reference to an environment. An example of intrinsic irreversibility is kaon decay.

Intrinsic irreversibility is of prime interest to Bohm and his collaborators, as well as to Prigogine's Brussels–Austin Group, because these types of irreversible processes are related to arrows of time. HS QM cannot give a rigorous description of these types of physical processes. One reason is that no HS elements exist whose survival probability has the right form of exponential decay:

$$P_s = |(\phi, e^{-iHt}\phi)|^2 \propto e^{-\Gamma t}. \quad (6)$$

It might be objected that physical systems decay with deviations from exponential decay which are too small to be measured experimentally. After all probabilities like P_s are observed in the laboratory as ratios of large numbers $N(t)/N(0)$, where $N(t)$ is the number of counts of a detector and $N(0)$ is the total population under observation. Unless the predicted deviations from (6) are of time scales comparable to $1/\Gamma$, they cannot be empirically observed. Recently deviations from exponential decay over short time scales for atoms undergoing quantum tunneling have been reported (Wilkinson *et al.*, 1997).

When decaying states are involved, (5) can be rewritten as

$$|\phi\rangle = \sum_n^N |\psi_n^G\rangle \langle\psi_n^G|\phi\rangle + \int |E\rangle \langle E|\phi\rangle d\mu(E) \quad (7)$$

where $|\phi\rangle$ represents the prepared state vector and $|\psi_n^G\rangle$, the so-called Gamow vector,⁵ represents decaying states. The first term on the rhs of (7) represents a sub-domain of the decaying state (note these are not elements of \mathcal{H} , hence no rounded brackets). There is usually only a small number N at the available scattering energies. The second term represents the background integral. The standard Weisskopf–Wigner approximation amounts to ignoring the background integral, but the preparation process does not always make this term negligible. The background integral *does not* have exponential time behavior, so if this term is substantial, deviations

⁵ As originally introduced, Gamow vectors were problematic in HS: their position probability density increased exponentially for large negative values of t . But since decay processes must begin at some past time $t = 0$, a RHS removes this physically problematic feature by allowing for more realistic boundary conditions (Bohm *et al.*, 1997).

from exponential decay will result. An effect of the background amplitude is often observed in resonance scattering experiments, whereas for decaying states, it is often neglected (Bohm, 1994). So observations of Wilkinson *et al.* can be explained as the effect of the background integral in (7). Their experiment involved a series of interventions (preparations) in the form of varying electromagnetic potentials to introduce variations in the acceleration of the atoms under observation leading to an *extrinsically* irreversible decay process.

Another reason for which intrinsic irreversibility cannot naturally be described in HS is that HS evolution is given by

$$U(t)|\phi(0)\rangle = e^{-iHt}|\phi(0)\rangle \tag{8}$$

where $U(t)$ is a unitary group generated by the Hamiltonian H for the system. The operator $U(t)$ is a continuous operator with respect to the topology $\tau_{\mathcal{H}}$ and forms a one-parameter group of operators. The inverse is defined as $U^{-1}(t) = U(-t)$ for all $-\infty < t < \infty$, so the evolution governed by $U(t)$ is time symmetric. However, semigroup operators lack an inverse. Therefore semigroups of operators are the appropriate operators for the evolution of intrinsically irreversible processes. In HS we must appeal to interactions with an environment (i.e. extrinsic irreversibility), whereas in RHS semigroup evolution and intrinsic irreversibility naturally arise.

If $U(t)$ is a unitary operator on \mathcal{H} and $\Phi \subset \mathcal{H} \subset \Phi^\times$, then U can be *extended* to Φ^\times provided that (1) U leaves Φ invariant, i.e. $U: \Phi \rightarrow \Phi$, and (2) U is continuous on Φ with respect to the topology τ_Φ . The operator U^\times denotes the *extension* of the HS operator U to Φ^\times and is defined by $\langle U\phi|F \rangle = \langle \phi|U^\times F \rangle$ for all $\phi \in \Phi$ and $F \in \Phi^\times$. Additionally $U^o: \Phi \rightarrow \Phi$ denotes the *restriction* of the HS operator U to Φ .

4. SCATTERING

Bohm and his co-workers have studied simple scattering experiments using RHS (e.g. Bohm *et al.*, 1997). Consider an accelerator which prepares a projectile and target in a particular state. The free particle Hamiltonian is H_o while the potential in the interaction region is V . The total Hamiltonian modeling the interaction of the particle with the target is, therefore, $H = H_o + V$.

An important step in their analysis of scattering experiments is the invocation of the *preparation/registration* arrow of time (Bohm *et al.*, 1994). The key intuition behind this arrow is that no observable properties can be measured in a state until the state has been prepared. Since it makes no sense to speak of a measurement of an observable such as the scattering angle until there is a state prepared by the accelerator. The time $t = 0$ marks the moment in time at which the state preparation is completed and the registration of detector counts can begin (any detector counts before this time must be discarded as noise). One of the consequences of the preparation/registration arrow is that some mathematical operations definable in

HS are nonsensical. For example one can calculate nonzero expectation values for an observable for $t < 0$, meaning that an observable has a nonzero expectation value before the state has been prepared.

Following Ludwig (1983, 1985) and Bohm *et al.* (1997), an in-state of a particular quantum system (conceived of as an ensemble of individual systems such as each elementary particle) is prepared by a preparation apparatus (a macrophysical system). The detector (considered to be classical) registers the post-interaction particles, also called out-states. In-states are taken to be elements $\phi \in \Phi_-$ and observables are taken to be elements $\psi \in \Phi_+$ (Decaying states, such as the Dirac, Lippman, Schwinger kets and Gamow vectors, are elements of Φ_+^\times).

The need to distinguish between states and observables implies the need for two RHS's, one for the states and one for the observables. The RHS $\Phi_- \subset \mathcal{H} \subset \Phi_-^\times$ is physically interpreted as the space of states while the RHS $\Phi_+ \subset \mathcal{H} \subset \Phi_+^\times$ is physically interpreted as the space of observables. The justification for these interpretations is as follows. The preparation/registration arrow implies the mathematical conditions $\int \langle E | \psi(t) \rangle dE = 0$ for all $t < 0$ and $\int \langle E | \phi(t) \rangle dE = 0$ for all $t > 0$. The requirements of analytic continuation leads naturally to a set of mathematical spaces fulfilling these conditions: Φ_- is the Hardy space of the lower complex energy half-plane intersected with the Schwartz class functions and Φ_+ is the Hardy space of the upper complex energy half-plane intersected with the Schwartz class functions (e.g. Bohm *et al.*, 1997).

For the space of states Φ_- , we seek a continuous evolution operator $U_-^o : \Phi_- \rightarrow \Phi_-$. U restricted to Φ_- fulfils this condition (i.e., it is continuous in τ_{Φ_-}), but only for $t \leq 0$. U_-^o carries states into the forward direction of time. Whereas U forms a unitary group on HS, its restriction to the domain Φ_- is a semigroup for times $t \leq 0$. Since U_-^o is τ_{Φ_-} -continuous for times $t \leq 0$, the extension of U to Φ_-^\times exists as a semigroup for $t \leq 0$.

Similarly for the space of observables Φ_+ , we seek a continuous evolution operator $U_+^o : \Phi_+ \rightarrow \Phi_+$. U restricted to Φ_+ fulfils this condition (i.e., it is continuous in τ_{Φ_+}) only for $t \geq 0$ and its temporal direction carries observables into the forward direction of time. Whereas U is unitary on HS, its restriction to the domain Φ_+ is a semigroup for times $t \geq 0$. Since U_+^o is τ_{Φ_+} -continuous for times $t \geq 0$, the extension of U to Φ_+^\times exists as a semigroup for $t \geq 0$.

U extended to Φ_-^\times and U extended to Φ_+^\times form two semigroups for which replacement of t with $-t$ is not defined. These semigroups fall out of the analysis quite naturally in the RHS framework providing a rigorous description of irreversible behavior in a scattering experiment (Bohm *et al.*, 1997).⁶

Some elements of the generalized eigenstates in Φ_-^\times and Φ_+^\times correspond to exponentially growing and decaying states respectively (Gadella, 1983; Bohm and

⁶In general $\Phi_- \neq \Phi_+$, but $\Phi_- \cap \Phi_+ \neq \{0\}$, so the semi-groups derived in this framework cannot be considered as leading to two disjoint families of eigenfunctions.

Gadella, 1989). The semigroups governing these states are

$$\langle \phi | U^\times | Z_R^* \rangle = e^{-iE_R t} e^{\frac{\Gamma}{2} t} \langle \phi | Z_R^* \rangle t \leq 0 \tag{9a}$$

$$\langle \psi | U^\times | Z_R \rangle = e^{-iE_R t} e^{-\frac{\Gamma}{2} t} \langle \psi | Z_R \rangle t \geq 0, \tag{9b}$$

where states $\phi \in \Phi_-$, observables $\psi \in \Phi_+$, E_R represents the total resonance energy, Γ represents the resonance width, Z_R represents the pole at $E_R - i\frac{\Gamma}{2}$, Z_R^* represents the pole at $E_R + i\frac{\Gamma}{2}$, $|Z_R^*\rangle \in \Phi_-^\times$ represents the growing Gamow vector and $|Z_R\rangle \in \Phi_+^\times$ represents the decaying Gamow vector. The $t < 0$ semigroup is identified as future-directed along with $|Z_R^*\rangle$ as forming/growing states. The $t > 0$ semigroup is identified as future-directed along with $|Z_R\rangle$ as decaying states.

The preparation/registration arrow plays a crucial role in these identifications, since it serves to specify the temporal direction of the semigroups. The space of functions plus the semigroup property alone are insufficient to determine the temporal direction of the semigroups. One can object that relying on notions of preparation and registration are operational or interventionist. Such an objection points to the good news/bad news nature of Bohm and colleagues' work. The good news is that, given the highly constrained context of the laboratory, operational procedures for preparations and registrations can be spelled out precisely. Such an approach seems justifiable for scattering experiments. The bad news is that the approach does not generalize straightforwardly to contexts outside the laboratory.

5. SEMIGROUPS IN THE BRUSSELS–AUSTIN APPROACH

Prigogine and co-workers have also analyzed scattering and decay experiments in their recent work. In their discussion of the Friedrichs model for scattering and resonance phenomena, Antoniou and Prigogine apply the RHS framework and show that the Hardy class functions form a natural function space for the analysis of quantum scattering and decay phenomena (Antoniou and Prigogine, 1993). Unlike in Bohm's approach, however, they do not make Ludwig's distinction between states and observables. Furthermore Antoniou and Prigogine adopt the following time-ordering: excitations are interpreted as events taking place before $t = 0$ while de-excitations are to be interpreted as events taking place after $t = 0$. This arrow is a kind of generalization of the preparation/registration arrow, but is based on observations rather than interventions.

The Brussels–Austin Group discusses two semigroups of evolution operators acting on states in Φ^\times . They split the test function space into two spaces Φ_- and Φ_+ based on their time-ordering rule. On reaching the point where choices have to be made regarding interpreting the directions of integration around the poles in the upper and lower complex half-planes for the Hardy class functions, they make the following choices: excitations (e.g. transitions from the continuum to the

eigenstate in the Friedrichs model, or formation of unstable states) are considered as past-oriented and are associated with contours in the upper half-plane, while de-excitations (e.g. mode–mode transitions in the Friedrichs model, or decay of unstable states) are considered as future-oriented and are associated with contours in the lower half-plane.

The eigenvectors of decaying states are associated with a discrete pole in the continuum and are represented by elements in the dual spaces Φ_-^\times and Φ_+^\times (Antoniou and Prigogine, 1993). By the same continuity requirements as in Bohm's approach, the evolution operators split into two time domains yielding

$$\langle \phi_+ | U^\times | Z_R^* \rangle = e^{iE_R t} e^{\frac{\Gamma}{2}t} \langle \phi_+ | Z_R^* \rangle t < 0 \quad (10a)$$

$$\langle \phi_- | U^\times | Z_R \rangle = e^{-iE_R t} e^{-\frac{\Gamma}{2}t} \langle \phi_- | Z_R \rangle t > 0, \quad (10b)$$

where $\phi_+ \in \Phi_+$ and $\phi_- \in \Phi_-$. Note that the roles of the upper and lower Hardy class function spaces is reversed with respect to Bohm's approach. The Brussels–Austin Group identifies the $t < 0$ semigroup as evolving states into the past along with $|Z_R^*\rangle$ as decaying states, while the $t > 0$ semigroup evolves states into the future along with $|Z_R\rangle$ as decaying states.

As noted earlier, the space of functions plus the semigroup property alone are insufficient to determine the temporal direction of the semigroups. The Brussels–Austin Group uses consistency with both empirical observations, as well as the ability of systems to communicate with each other, in order to determine the directions of the semigroups (Antoniou and Prigogine, 1993; Antoniou, private communication).⁷

6. RELATING THE TWO APPROACHES

There are two immediate observations when comparing the work of Bohm and Brussels–Austin Groups: 1) The time directions identified for the $t < 0$ semigroups differ between the two research groups. 2) The roles of the Hardy class spaces are reversed. Both differences can be traced to the temporal arrows and contexts invoked in the two approaches.

Bohm envisions the case of a scattering experiment, where the preparation/registration arrow is built into the experimental arrangement by the very nature of the interventions required. In its most general form this arrow expresses the idea that observable properties do not exist apart from some physical state, i.e., observable properties logically presuppose states. However, the laboratory context accounts for the additional criteria allowing Bohm to identify the two semigroups as both being future-directed. Hence, there is a clearly motivated arrow of time, albeit in a limited context.

⁷ The approach for transient scattering can be extended to the case where the interactions are continuous and persistent, yielding similar results (Petrosky and Prigogine, 1997b).

Antoniou and Prigogine envision a more general situation where excitations and de-excitations of states occur in the absence of laboratory-type interventions. Invoking such an arrow of time along with other conditions (e.g. the ability to communicate) leads to the assignment of temporal directions in (10). The $t < 0$ semigroup is ignored as we never observe it, leading to a consistent description of irreversible processes. Nevertheless, the generality of the excitation/de-excitation arrow gives us no physically rigorous argument for the temporal arrow because the arrow is supplemented with conditions consistent with our experience.

One might think that Antoniou’s and Prigogine’s not distinguishing between states and observables leads to the differences between the two groups since $\Phi_- \subset \Phi_+$. However this is not the case, because the distinction between states and observables is dependent upon the preparation/registration arrow. Although preparations are particular kinds of excitations and registrations are associated with particular kinds of de-excitations, the Brussels–Austin focus on states leads naturally to a different splitting of the RHSs based on their more general arrow.

7. INTERPRETING THE RHS FORMALISM

Many advocates of RHS are reluctant to give a realistic interpretation to the elements of the mathematical framework. Bohm and the Brussels–Austin Group are likewise cautious in this regard, but indicate that they have some (possibly surreptitious) realist leanings regarding these elements. For example both groups consider the elements of Φ to represent possible physical states or observables of the system (Bohm, 1967; Antoniou and Prigogine, 1993).⁸

There are tensions, nevertheless, between what is considered realistic versus what is considered merely useful for computational purposes. For example Bohm, following Ludwig (1983, 1985), takes the preparation and registration apparatuses to be classical devices. Preparation apparatuses prepare the states ϕ while registration apparatuses detect or register the observables (or the values thereof) ψ . These observables are considered to be the “real” physical entities (Bohm *et al.*, 1997, pp. 496–497). Bohm, however, often asserts that microsystems—the “agents by which the preparation apparatus acts on the registration apparatus”—are imaginary (e.g. Bohm *et al.*, 1994, p. 443) or that there is no need to assume they exist (Bohm *et al.*, 1997, p. 496). The “imagined entities connected with microphysical systems are not restricted to Φ ; indeed their energy distributions do not have to be well-behaved functions of the energy” (Bohm *et al.*, 1997, p. 497). Therefore, Bohm concludes that, “for the hypothetical entities connected with microphysical systems, like Dirac’s ‘scattering states’ $|p\rangle$ or Gamow’s ‘decaying states’ $|E - i\Gamma/2\rangle$, the RHS formulation has a much larger choice and can describe them by elements of Φ^\times ” (Bohm *et al.*, 1997, p. 497; see also Bohm *et al.*, 1994).

⁸ Indeed although both Φ and \mathcal{H} are mathematical idealizations, Bohm takes Φ to be “closer” to reality than \mathcal{H} (Bohm, 1978, p. 21).

On the other hand, Bohm seems inclined at some points to attribute reality to these entities: “. . . there may be a larger class of ‘microphysical states’ (in addition to the Dirac Kets, Gamow vectors, virtual state vectors . . . and others . . .) which still await their physical interpretation” (Bohm *et al.*, 1994, p. 446).⁹ Or again, “Though there is an abundance of resonance states in nature which are described by first-order Gamow vectors and which evolve according to the exponential law, there is no direct experimental evidence for microphysical objects associated with N th order poles, $N > 1$, of the S-matrix which are described by higher-order Gamow vectors” (Bohm *et al.*, 1997, p. 529). His comparison of the evidence for objects described by first order Gamow vectors with the lack of evidence for objects described by higher-order Gamow vectors appears to assume a realistic construal of such entities. Indeed, he writes that the physical meaning of these higher-order Gamow vectors is questionable in contrast with those of first-order (Bohm *et al.*, 1997, p. 532). Bohm then continues, “ordinary Gamow states have been identified in abundance, e.g., through their Breit–Wigner profile in scattering experiments, or the exponential decay law” (Bohm *et al.*, 1997, p. 532).

Although rarely making interpretive comments, the Brussels–Austin Group also give indications of realist leanings. For example they assert that the eigenvalues of observables in Φ^\times “influence” the evolution and produces decay (Antoniou and Prigogine, 1993, p. 454) and suggest that resonances should be associated with physical observables in unstable systems (Antoniou and Melnikov, 1998). As well, they seek to reify distributions (elements of Φ^\times) as the fundamental ontological elements of descriptions in both classical and quantum unstable systems (Petrosky and Prigogine, 1997a,b; Bishop, 2004).

8. TOWARD A REALISTIC INTERPRETATION OF RHS QM

The RHS formalism has proven useful for illuminating our understanding of particular irreversible processes found in a variety of unstable systems (e.g. approach to equilibrium, decay, scattering). Yet the prospects for answering our general questions about irreversibility and the origin of various arrows of time are not clarified as yet. The preparation/registration arrow proposed by Bohm clarifies the nature of irreversibility in scattering experiments, but is much too limited for application to more general settings. For example when the restriction to laboratory interventions is dropped—as in the excitation/de-excitation arrow proposed by the Brussels–Austin Group—the arrow can no longer uniquely determine the direction of the evolution semigroups governing physical processes. We must still choose the temporal directions of the semigroups based on additional criteria such as observational experience or consistency.

⁹ Bohm’s point here is that there may be many other elements in the space Φ^\times that have a physical correspondence or interpretation (i.e. there may be new, as yet undescribed, physics), a point on which he was silent in earlier writings on RHS quantum mechanics.

Compared to the standard HS framework, the RHS framework provides a significant advantage in the description of irreversible processes in that semigroup evolutions arise naturally in the latter. Obviously more than the presence of semigroups is needed, however, in order to explain the arrow of time in quantum mechanics. One suggestion that could contribute to a more complete understanding is to develop a robust realist interpretation of the elements of the RHS formalism. A crucial reason why a realist interpretation may prove important to further clarifying irreversibility and the quantum arrow of time is provided by one of the core intuitions of the Bohm and Brussels–Austin approaches: namely, that irreversibility is rooted in the dynamics of physical systems. If that is the case, then the elements of the RHS formalism have to be mapped onto elements of physical systems. So the tensions discussed earlier need to be clarified and the realist suggestions filled out in order to better elucidate the dynamical mechanisms at work in irreversible processes.

A realist interpretation of the elements of the RHS formalism cannot be carried out generically (see Melsheimer, 1974a,b for some indications why). It requires concrete realizations of the dual pair $\{\Phi, \Phi^\times\}$ which are tied to the algebra of observables of the systems in question. Once such an interpretation for a given dual pair is in place, the power of the RHS framework for clarifying and illuminating the dynamical processes responsible for irreversible behavior of these systems should be greatly enhanced. Some evidence for this can be seen in (Bohm *et al.*, 1997; Petrosky and Prigogine, 1997a,b; Bishop, 2004).

For example in RHS QM, the decay of scattering states is associated with a Gamow vector with eigenvalue $\lambda = E_R - i\frac{\Gamma}{2}$, a mathematical element not well defined on HS. The Gamow vector involves physical quantities such as the resonance energy and the full width at half maximum (note that $\Gamma = 0$ corresponds to the rest energy for the composite particle). Furthermore, under the Bohm approach, the condition $\langle E|\psi(t)\rangle = 0$ for all $t < 0$ refers to the energy distribution of the detected state while $\langle E|\phi(t)\rangle = 0$ for all $t > 0$ refers to the incident beam resolution. All of these quantities are physically measurable; however, the concepts involved (energy, momentum, time, etc.) are not exhausted simply by associating them with preparation or measurement procedures. Hence, in the RHS framework, one can then make a direct correspondence between mathematical elements, on the one hand, and their physical counterparts and causal efficacy, on the other hand, in a way that goes beyond operational procedures.

Other viewpoints focus on initial conditions as the explanation for irreversibility and arrows of time. However, realistic initial conditions involved in explaining irreversibility and time arrows often cannot be formulated in HS. For example in the case of scattering, the standard initial condition in HS is that states are not interacting with the scattering center at $t \rightarrow -\infty$. Of course this initial condition is unrealistic as the particles crucial to the experiment have not been created or properly prepared until some finite time before the interaction. Yet HS cannot

accommodate more physically realistic initial conditions for scattering processes (Bohm *et al.*, 1997). The RHS framework can accommodate realistic initial conditions in a natural way; so a realistic interpretation of the elements of RHS could also play a fruitful role for the special initial conditions route to explanations of irreversibility as well.

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